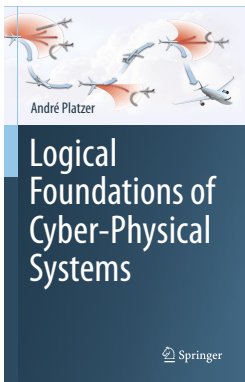


# 08: Events & Responses

## Logical Foundations of Cyber-Physical Systems



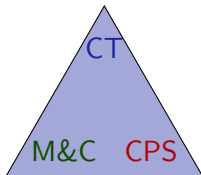
André Platzer



- 1 Learning Objectives
- 2 The Need for Control
  - Events in Control
  - Cartesian Demon
  - Event Detection
- 3 Event-Triggered Control
  - Evolution Domains Detect Events
  - Non-negotiability of Physics
  - Dividing Up the World
  - Event Firing
  - Physics vs. Control
  - Event-Triggered Verification
- 4 Summary

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using loop invariants  
design event-triggered control

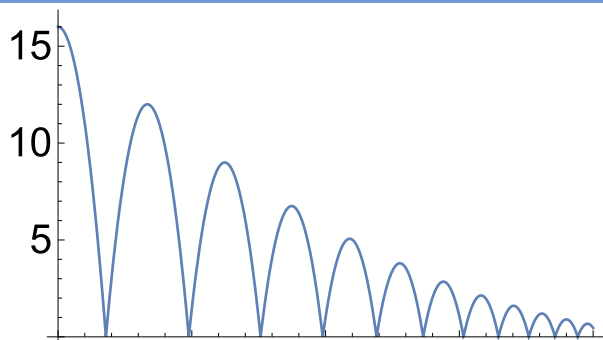


modeling CPS  
event-triggered control  
continuous sensing  
feedback mechanisms  
control vs. physics

semantics of event-triggered control  
operational effects  
model-predictive control



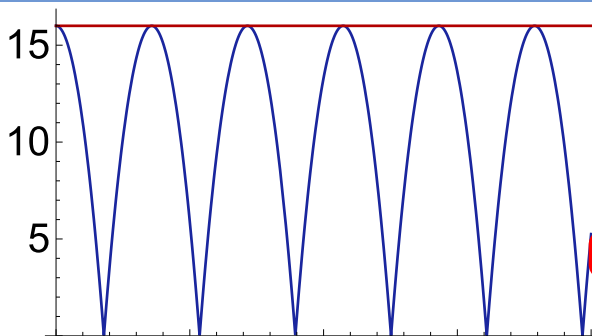
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Proposition (Quantum can bounce around safely)

$$0 \leq x \wedge x = H \wedge v = 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \rightarrow$$

$$[(\{x' = v, v' = -g \ \& \ x \geq 0\}; (?x=0; v := -cv \cup ?x \neq 0))^*](0 \leq x \wedge x \leq H)$$



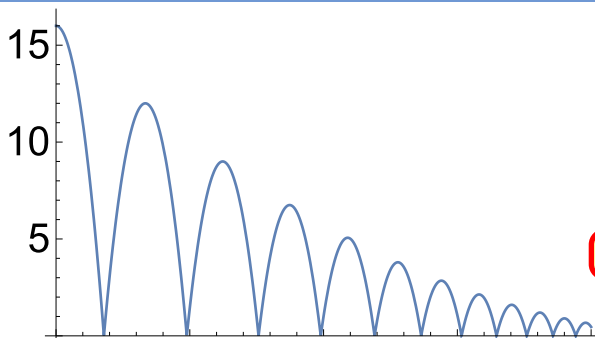
Can be improved...

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Proof **@invariant**( $2gx = 2gH - v^2 \wedge x \geq 0$ )



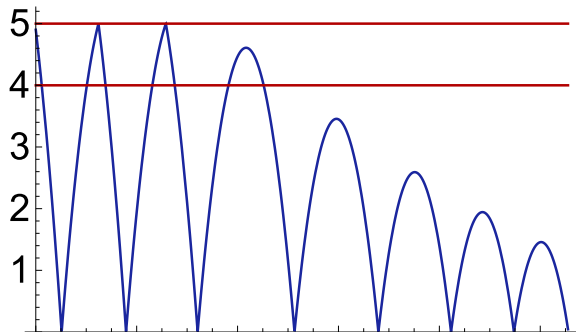
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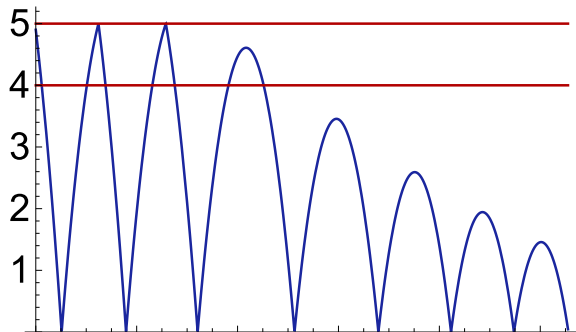
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

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# Quantum the Daring Ping-Pong Ball



Conjecture (Quantum can play ping-pong safely)

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Proof?

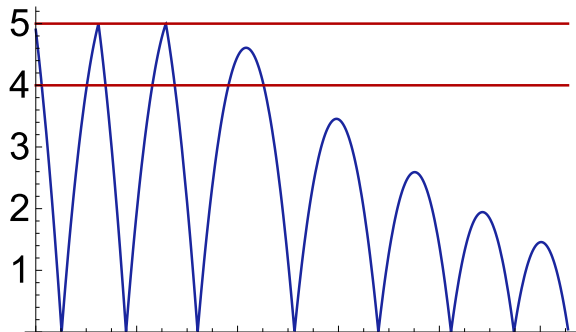
Ask René Descartes



## Outwit the Cartesian Demon

Skeptical about the truth of all beliefs until justification has been found.

# Quantum the Daring Ping-Pong Ball



Conjecture (Quantum can play ping-pong safely)

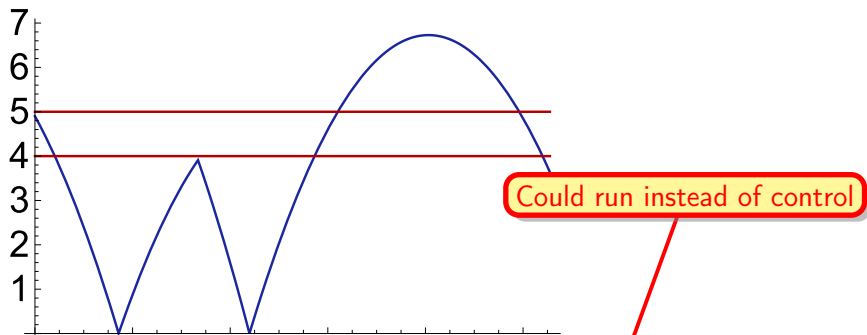
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Proof?

Ask René Descartes



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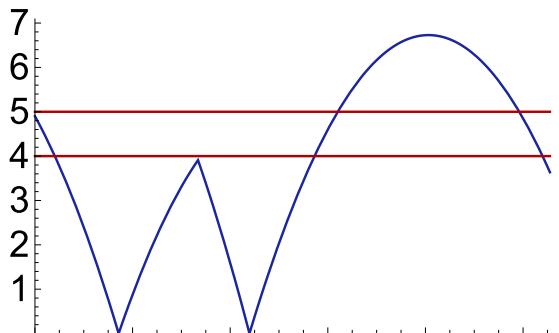
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Proof? Ask René Descartes who says no!

# Quantum the Daring Ping-Pong Ball



No bounce nor event

Conjecture (Quantum can play ping-pong safely)

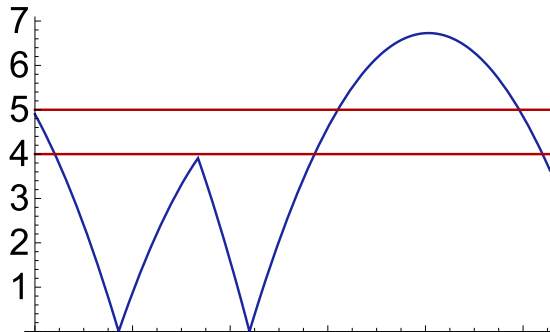
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Proof?

Ask René Descartes who says no!



Could miss this event

Conjecture (Quantum can play ping-pong safely)

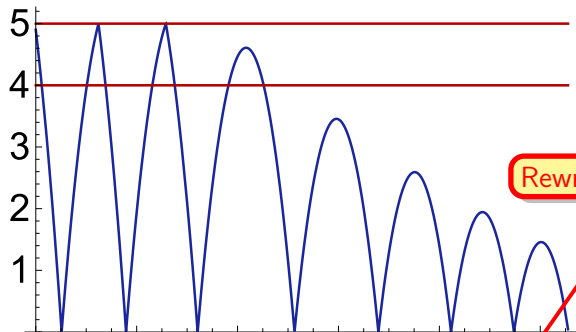
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Proof?

Ask René Descartes who says no!



Rewrite as if-then-else

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

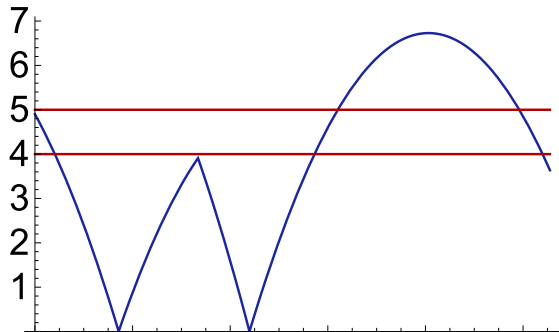
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Proof?

Ask René Descartes





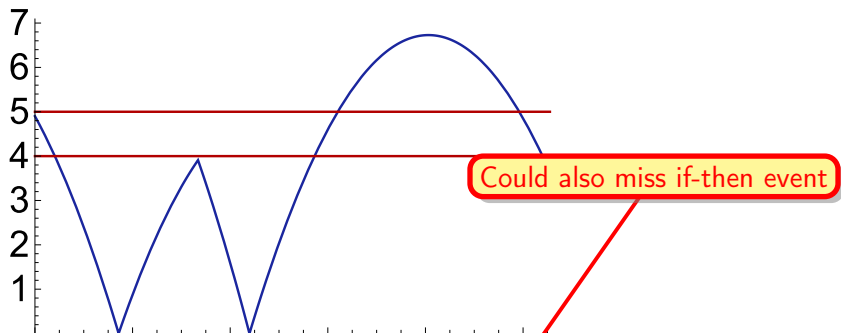
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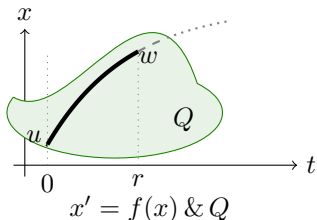
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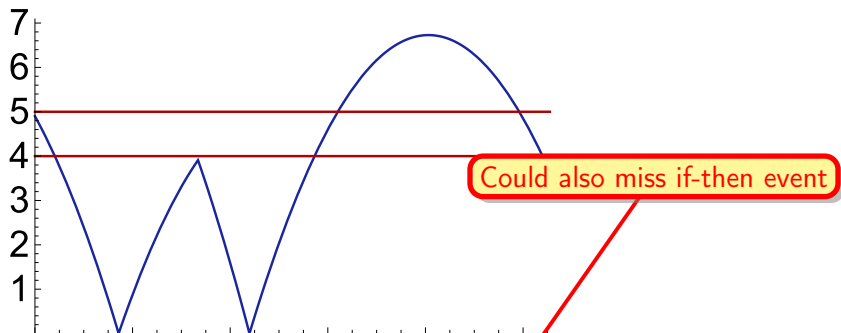
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## Evolution domains detect events

$$x' = f(x) \ \& \ Q$$

Evolution domain  $Q$  of a differential equation is responsible for detecting events.  $Q$  can stop physics whenever an event happens on which the control wants to take action.





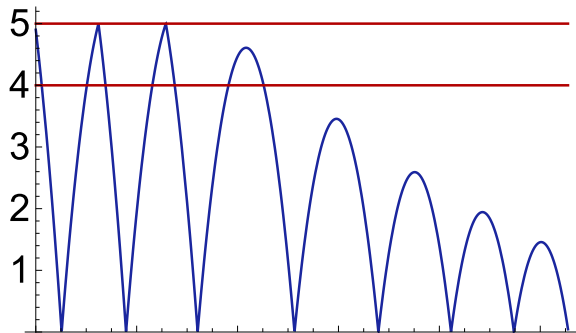
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Proof? Ask René Descartes who says no!



Domain as event trap?

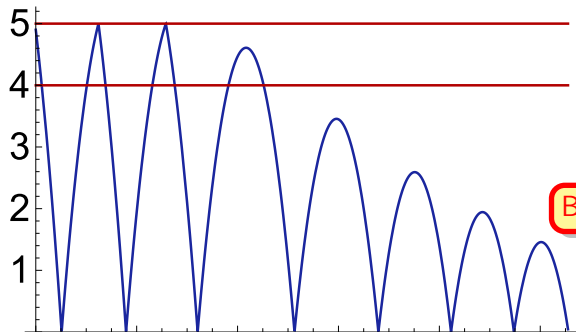
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Proof? Ask René Descartes who says no!



Broken physics: Always event

Conjecture (Quantum can play ping-pong safely)

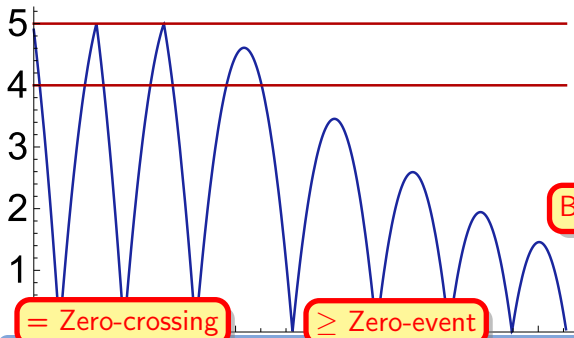
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# Quantum the Deterministically Daring Ping-Pong Ball

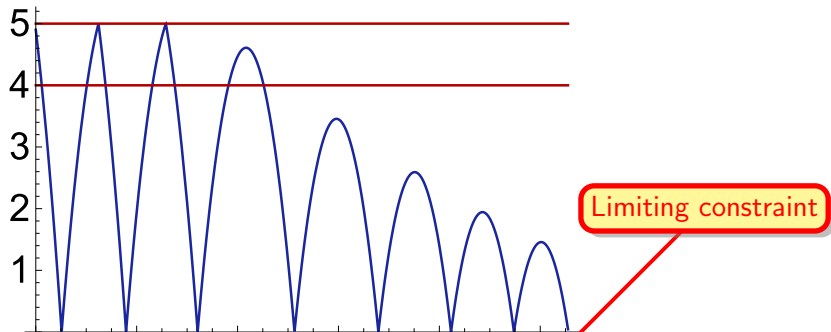


Conjecture (Quantum can play ping-pong safely)

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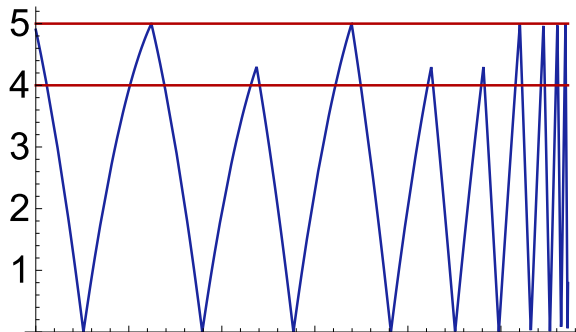
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Proof?

Ask René Descartes



May miss 4 but not 5

Conjecture (Quantum can play ping-pong safely)

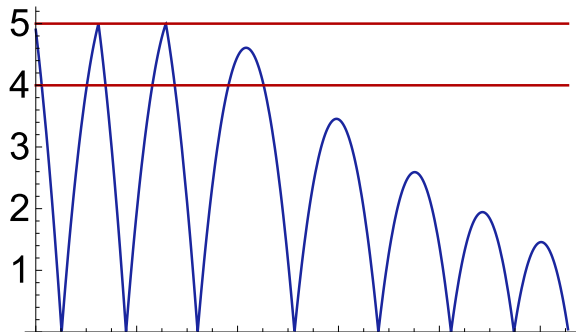
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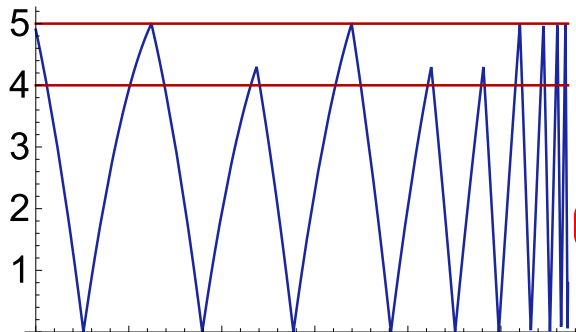
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Proof? Ask René Descartes who says yes!



Domain by construction

Conjecture (Quantum can play ping-pong safely)

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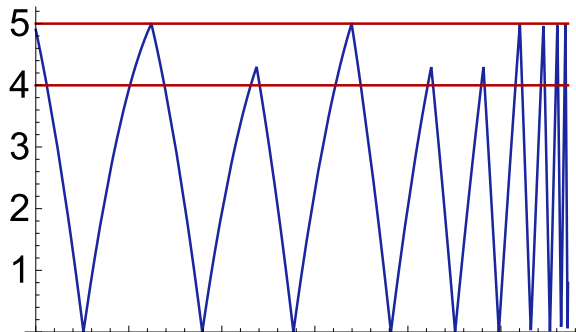
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Proof?

Ask René Descartes who says yes! But meant to say no!

# Quantum the Deterministically Daring Ping-Pong Ball



Non-negotiable physics

Conjecture (Quantum can play ping-pong safely)

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Proof?

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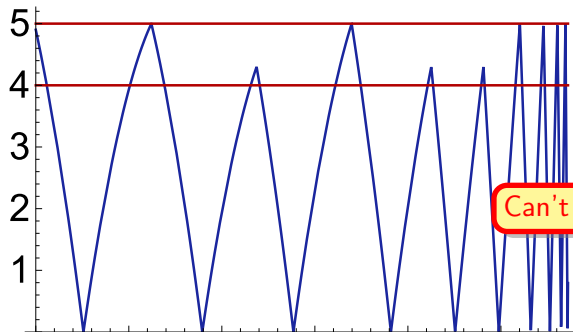
## Non-negotiability of Physics

- 1 Making systems safe by construction is a great idea. For control!
- 2 But not by changing the laws of physics.
- 3 Physics is unpleasantly non-negotiable.
- 4 If models are safe because we forgot to include all behavior of physical reality, then correctness statements only hold in that other universe.

Despite control

We don't get to boss physics around

We don't make this world any safer by writing CPS programs for another universe.



Can't stop the world for an event

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

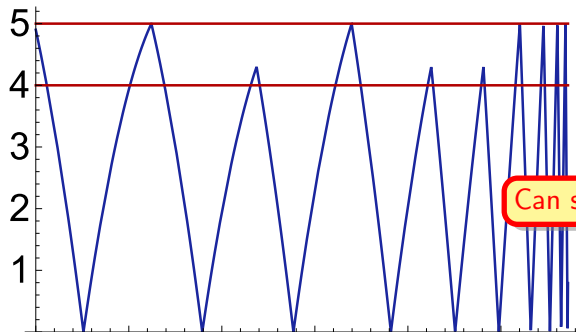
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Proof?

Ask René Descartes who says yes! But meant to say no!

# Quantum the Deterministically Daring Ping-Pong Ball



Can split the world for an event

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

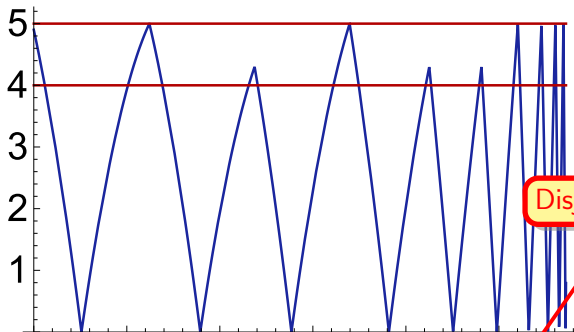
$$[(((\{x' = v, v' = -g \ \& \ x \geq 0 \wedge x \leq 5\} \cup \{x' = v, v' = -g \ \& \ x > 5\});$$

$$\text{if}(x=0) \ v := -cv \ \text{else if}(4 \leq x \leq 5) \ v := -fv)^*](0 \leq x \leq 5)$$

Proof?

Ask René Descartes





Disjoint domains

Shattered the world

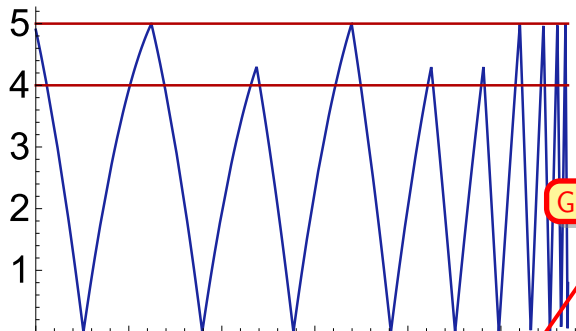
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Proof? Ask René Descartes



Glue domains

Reunite the world

Conjecture (Quantum can play ping-pong safely)

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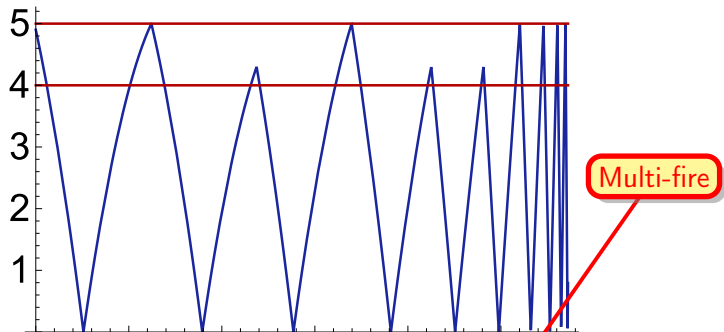
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Proof? Ask René Descartes

## Connected evolution domains

- 1 Evolution domain constraints need care.
  - 2 Determine regions within which the system can evolve.
  - 3 Disconnected/disjoint disallows continuous transitions.
- 
- 1 Splitting the state space into different regions to detect events is fine.
  - 2 Destroying the world is not.
  - 3 Not even by poking infinitesimal holes into the time-space continuum.





Conjecture (Quantum can play ping-pong safely)

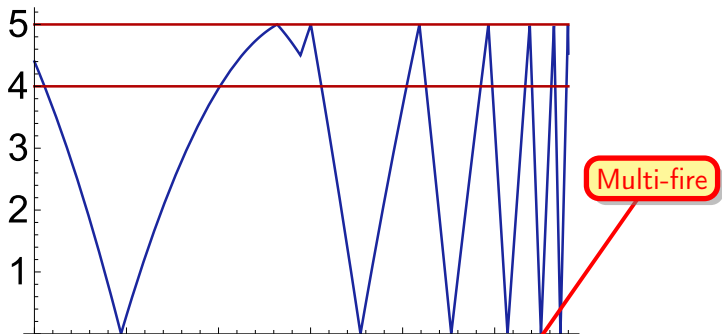
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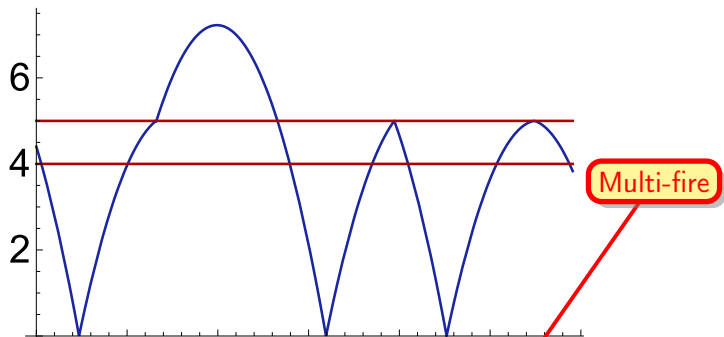
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Proof?

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Conjecture (Quantum can play ping-pong safely)

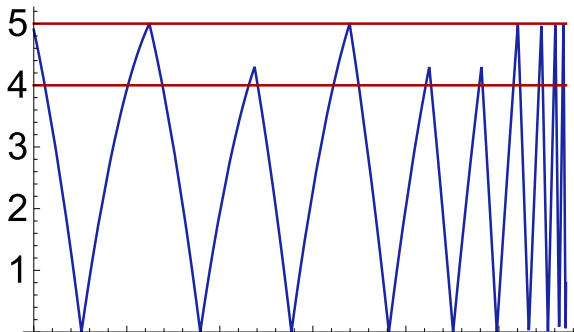
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$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5) v := -fv)^*](0 \leq x \leq 5)$$

Proof?

Ask René Descartes who definitely says no!



Only upsense event

Conjecture (Quantum can play ping-pong safely)

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$$[(((\{x' = v, v' = -g \ \& \ x \geq 0 \wedge x \leq 5\} \cup \{x' = v, v' = -g \ \& \ x \geq 5\});$$

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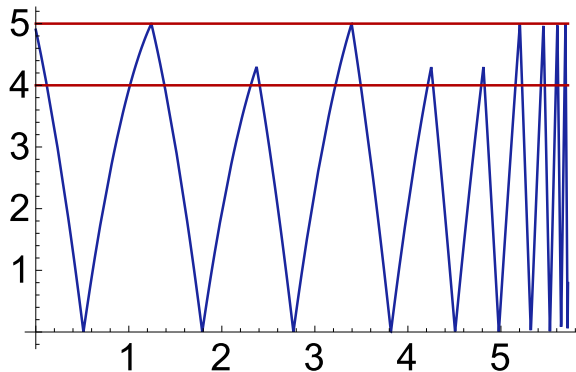
Proof?

Ask René Descartes



## Multi-firing of events

- 1 If the same event is detected multiple times:
- 2 Are multiple responses acceptable?
- 3 Or is a single response crucial?



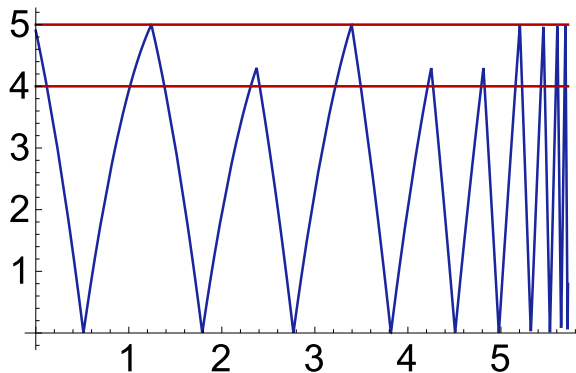
**control:** robust, all cases  
**physics:** precise

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[[({x' = v, v' = -g \& x \geq 0 \wedge x \leq 5}) \cup \{x' = v, v' = -g \& x \geq 5\}];$$

$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) v := -fv)^*](0 \leq x \leq 5)$$



**control:** robust, all cases  
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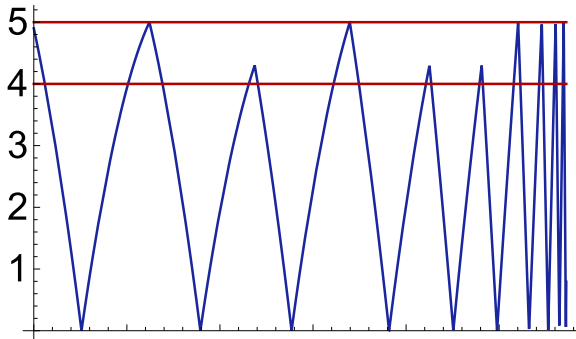
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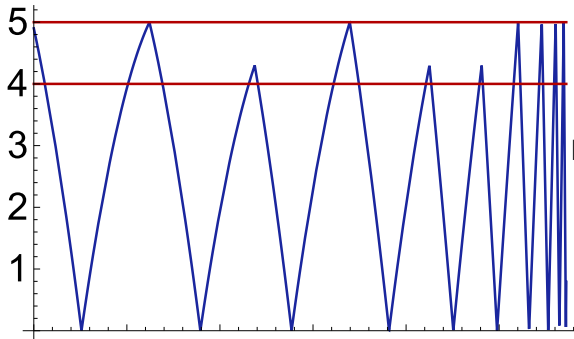
# Quantum's Ping-Pong Proof Invariants

Proposition (▶ Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[\{x' = v, v' = -g \ \& \ x \geq 0 \wedge x \leq 5\} \cup \{x' = v, v' = -g \ \& \ x \geq 5\}];$$

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Loop invariant  $j(x, v)$ :









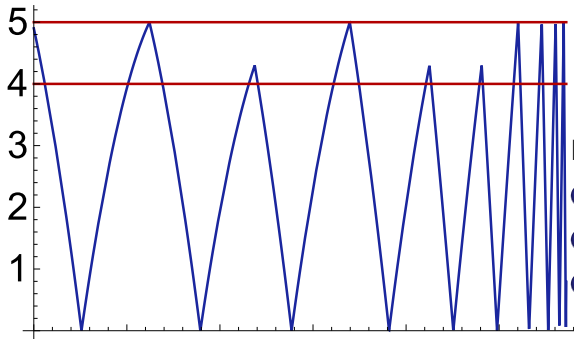


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$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

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$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) v := -fv]^* (0 \leq x \leq 5)$$



Loop invariant  $j(x, v)$ :

- ①  $0 \leq x \leq 5$  not inductive
- ②  $0 \leq x \leq 5 \wedge v \leq 0$  not inductive
- ③  $0 \leq x \leq 5 \wedge (x=5 \rightarrow v \leq 0)$

# Quantum's Ping-Pong Proof Invariants

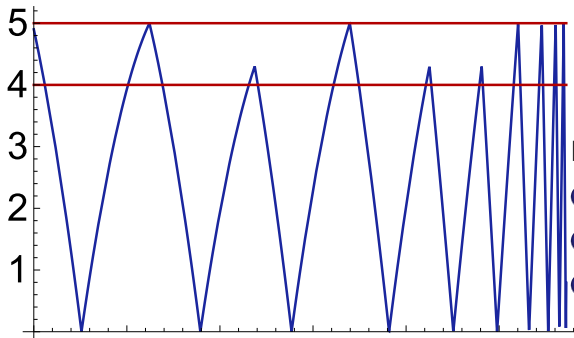
Proposition (▶ Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[\{x' = v, v' = -g \wedge x \geq 0 \wedge x \leq 5\} \cup \{x' = v, v' = -g \wedge x \geq 5\}];$$

$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) v := -fv^*] (0 \leq x \leq 5)$$

Proof @invariant( $0 \leq x \leq 5 \wedge (x = 5 \rightarrow v \leq 0)$ )



Loop invariant  $j(x, v)$ :

- ①  $0 \leq x \leq 5$  not inductive
- ②  $0 \leq x \leq 5 \wedge v \leq 0$  not inductive
- ③  $0 \leq x \leq 5 \wedge (x = 5 \rightarrow v \leq 0)$  yes!

# Quantum's Ping-Pong Proof Invariants

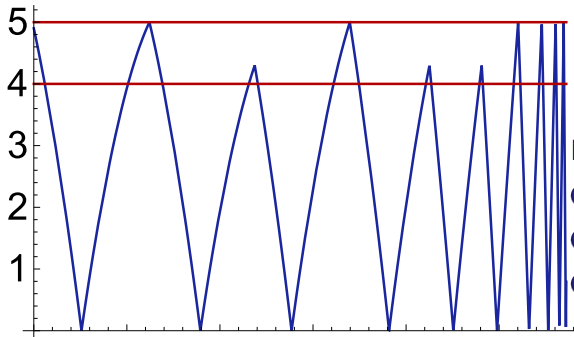
Proposition (▶ Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

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Proof **@invariant**( $0 \leq x \leq 5 \wedge (x = 5 \rightarrow v \leq 0)$ )



Just can't implement ...

Loop invariant  $j(x, v)$ :

- 1  $0 \leq x \leq 5$  not inductive
- 2  $0 \leq x \leq 5 \wedge v \leq 0$  not inductive
- 3  $0 \leq x \leq 5 \wedge (x = 5 \rightarrow v \leq 0)$  yes!

- 1 Learning Objectives
- 2 The Need for Control
  - Events in Control
  - Cartesian Demon
  - Event Detection
- 3 Event-Triggered Control
  - Evolution Domains Detect Events
  - Non-negotiability of Physics
  - Dividing Up the World
  - Event Firing
  - Physics vs. Control
  - Event-Triggered Verification
- 4 Summary

- 1 One important principle for designing feedback mechanisms
- 2 Conceptually simple: detect all relevant events and respond correctly
- 3 Assumes all events are surely detected
- 4 Implementation: Requires continuous sensing  
Tell me if you ever find a faithful implementation platform ...
- 5 Robust events, not just:  $\text{if}(x = 9.8696)$  ...
- 6 Events have subtle models, but make design and verification easier!  
Non-negotiability of Physics   Connected domains   Multi-firing ●
- 7 Useful abstraction when system evolves slowly but senses quickly
- 8 Verify event-triggered model as first step
- 9 Then refine toward realistic implementation based on safe event-triggered design
- 10 Physics  $\neq$  Control

## Non-negotiability of Physics

- 1 Making systems safe by construction is a great idea. For control!
- 2 But not by changing the laws of physics.
- 3 Physics is unpleasantly non-negotiable.
- 4 If models are safe because we forgot to include all behavior of physical reality, then correctness statements only hold in that other universe.

Despite control

We don't get to boss physics around

We don't make this world any safer by writing CPS programs for another universe.



André Platzer.

*Logical Foundations of Cyber-Physical Systems.*

Springer, Switzerland, 2018.

URL: <http://www.springer.com/978-3-319-63587-3>,

doi:10.1007/978-3-319-63588-0.



André Platzer.

*Logical Analysis of Hybrid Systems: Proving Theorems for Complex Dynamics.*

Springer, Heidelberg, 2010.

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Differential refinement logic.

In Martin Grohe, Eric Koskinen, and Natarajan Shankar, editors, *LICS*, pages 505–514, New York, 2016. ACM.

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