SAFE DIVING

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OUTLINE

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5. Heart rate determines VO2 consumed
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MOTIVATION

- There are over 1,000 cases of Decompression Sickness (DCS) per year
- Many cases are due to lack of awareness of remaining air supply
- Prevent DCS and Arterial Gas Embolism
- Most computers do not display remaining dive time, the ones that do cost over $2,000

RELATED WORK

- US Navy most knowledgable body on SCUBA diving
  - Almost all of their results come from testing
- Implantable Cardiac Medical Devices
  - Verification performed via clinical studies
- Haque et al. noted a lack of verification systems for CPS

MODEL*

Velocity —determines (1)→ Heart Rate —determines (2)→ VO2 cons.

* Time Triggered

SAFETY

$O2 \text{ in tank} >= depth / v_{ascent} * VO2_{max}$

more conservative than $VO2_{ascent}$

1. VELOCITY $\xrightarrow{\text{determines}}$ HEART RATE

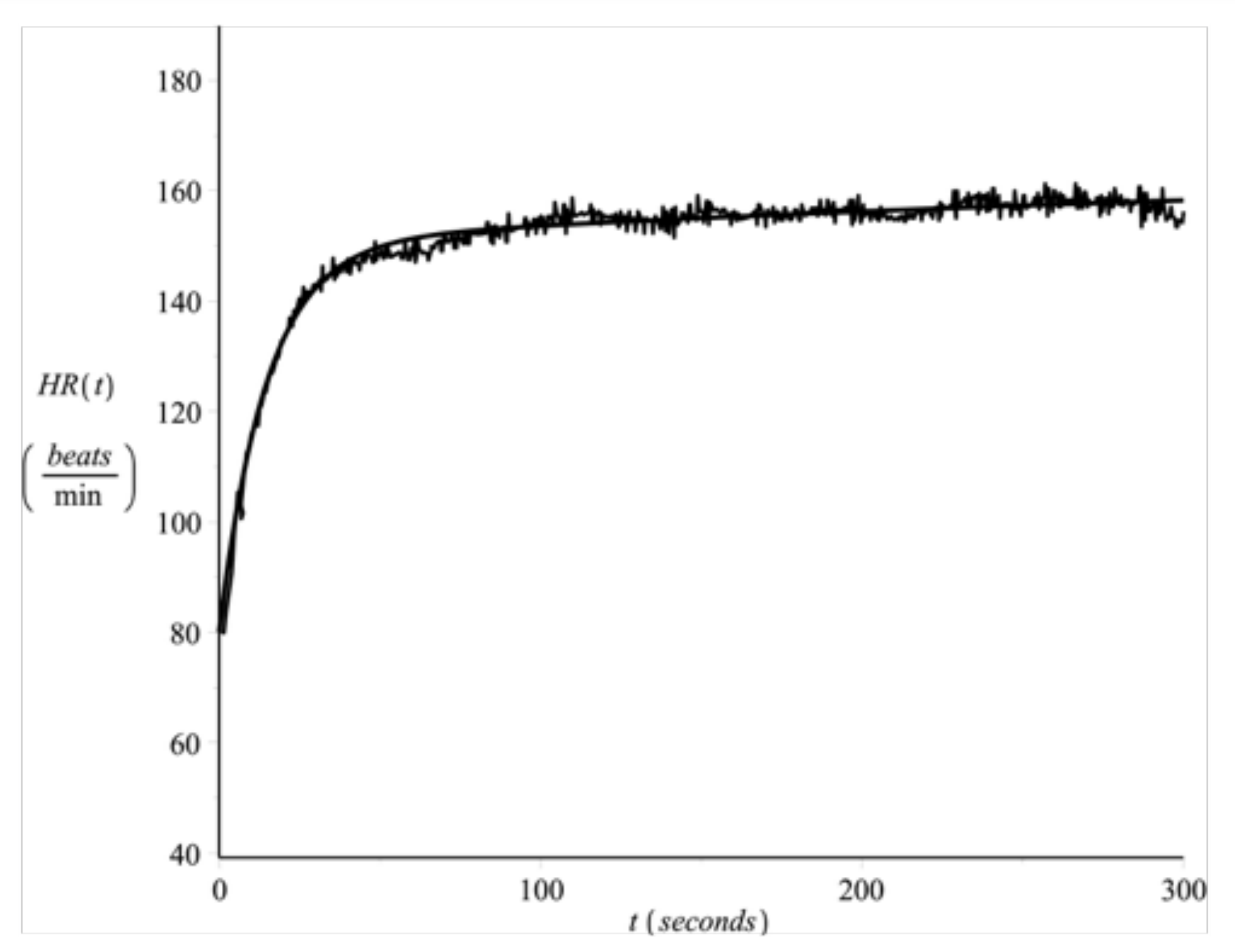
$$HR' = f_{\min} \cdot f_{\max} \cdot f_d$$

$$f_{\min}(HR) = 1 - e^{\left(\frac{HR - HR_{\min}}{stdev}\right)^2}$$

$$HR_{\min} = \frac{35}{cond} \frac{beats}{\min}$$

$$f_{\max}(HR) = e^{\left(\frac{HR_{\max} - HR}{stdev}\right)^2} - 1$$

$$HR_{\max} = 200 \frac{beats}{\min}$$



1. VELOCITY $\xrightarrow{\text{determines}}$ HEART RATE

$$\boxed{f_d(HR,v,t) = -\alpha \cdot cond \cdot (HR - L(v,t))}$$

$$L(v,t) = \alpha_3 \cdot L_{cond}(v) \cdot L_t(t)$$

$$L_{cond}(v) = L_{base} + (L_{\max} - L_{base}) \cdot e^{\alpha_6 (v - v_{\max})}$$

$$L_t(t) = 1 - e^{-\frac{t}{\alpha_7}}$$

$$\alpha = 0.08 s^{-1}$$

$$\alpha_3 = 4 beats \cdot \min^{-1} \cdot mM$$

$$L_{base} = 1 mM$$

$$L_{\max} = 9 mM$$

$$\alpha_6 = 1.8$$

$$v_{\max} = 8.88\sqrt{cond}$$

$$\alpha_7 = 2700 s^{-1}$$

PROBLEM?

- TOO COMPLICATED
  - Multiple exponential terms in the differential equation — ‘e’ not in the syntax for d$\mathcal{L}$
  - No algebraic intuition of solution — author numerically fit the data for runners

SOLUTION?

- SIMPLIFY !
- Aspects to replicate in simplified dynamics:
    - intensity (velocity) should determine max heart rate achieved (steady state)
    - velocity should affect time taken to reach that steady state
    - “reach” = come within 1 beat/min of steady state

SOLUTION

$$HR = HR_{ss} - (HR_{ss} - HR_{init})e^{-\frac{t\ln(HR_{ss}-HR_{init})}{(-20\frac{v}{v_{\max}}+30)}}$$

Steady State: $$HR_{ss} = HR_{\min} + \frac{v}{v_{\max}}(HR_{\max} - HR_{\min})$$

Time scale: $$\frac{\ln(HR_{ss}-HR_{init})}{(-20\frac{v}{v_{\max}}+30)}$$

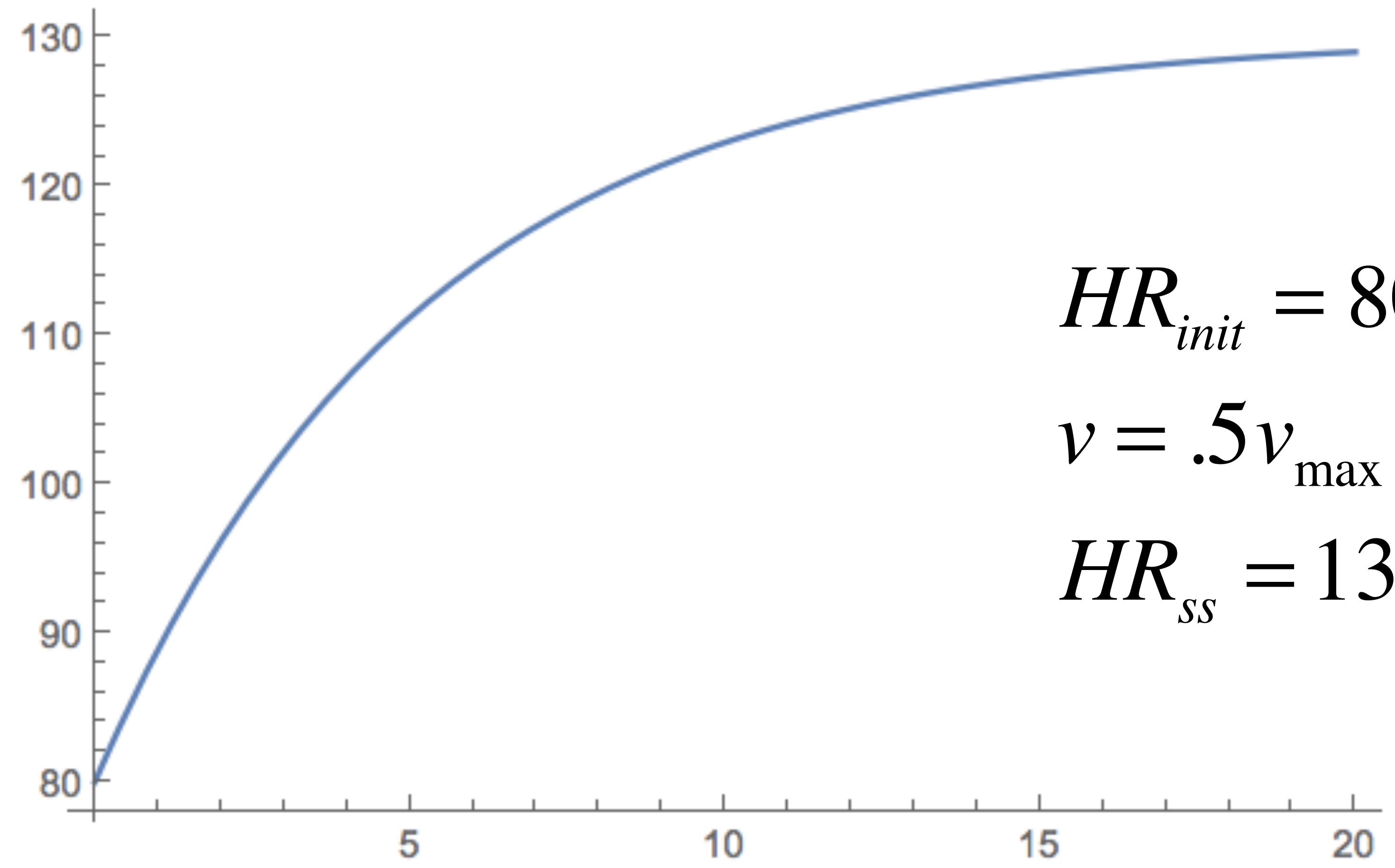
As $v \to v_{\max}$, $HR \to HR_{ss} - 1$

Good approximation.
usually within 1% of HR_{ss}

$$HR = HR_{ss} - (HR_{ss} - HR_{init})e^{-\frac{t\ln(HR_{ss}-HR_{init})}{(-20\frac{v}{v_{\max}}+30)}}$$

```
In[2]:= Plot[x = 130 - 50*Exp[-t*Log[50]*1/20], {t, 0, 20}]
```

Out[2]=



$$HR_{init} = 80bpm$$

$$v = .5v_{\max}$$

$$HR_{ss} = 130bpm$$

DYNAMICS

$$HR' = -(HR - HR_{ss})\frac{c}{-20(\frac{v}{v_{\max}}) + 30}$$

$$2.71828^{c} = HR_{ss} - HR_{init}$$

OK to approximate **e** because:

- c is a constant, not part of dynamics
- computer also approximates value of e

2. HEART RATE $\xrightarrow{\text{determines}}$ VO2 CONS

$$\%HR_{max} = 0.6463 * \%VO2_{max} + 36.8$$

$$VO2_{max} = 60 \frac{ml}{kg * \min} = 288 \frac{l}{\min}$$ (for 80kg diver)

$$HR_{max} = 200bpm$$

$$VO2 = \frac{VO2_{max}}{0.6463 * HR_{max}} * HR - \frac{36.8}{0.6463}$$

$$VO2' = \frac{VO2_{max}}{0.6463 * HR_{max}} HR'$$

FINAL VO2 DYNAMICS

$$VO2' = \frac{VO2_{\max}}{0.6463 * HR_{\max}}\left(-(HR - HR_{ss})\frac{c}{-20\left(\frac{v}{v_{\max}}\right) + 30}\right)$$

CONTROLLER

```
{
    v:=*;
    {
        ?(v = vAsc);         /* velocity should be = vAsc during ascent */
        depthV := -1*v;
    }
    ++
    {
        ?(Tank - VO2max*T >= depth/vAsc * VO2max); /*guard for case with no change in depth*/
        ?(v <= vMax & v >= 0);
        depthV := 0*v;
    }
    ++
    {
        ?(depth + v*T <= depthMax); /* guard on v to ensure we don't go below maximum depth in T */
        ?(Tank - VO2max*T >= (depth - vMax*T)/vAsc * VO2max); /* guard on case with increasing depth */
        ?(v = vAsc);         /* velocity should be = vAsc during decent */
        depthV := 1*v;
    }
}
```

INVARIANT

$$HR_{\min} <= HR <= HR_{\max}$$

$$Tank >= \frac{depth}{v_{asc}} * VO2_{\max}$$

$$VO2 <= VO2_{\max}$$

$$v < v_{\max}$$

$$depth < depth_{\max}$$

$$HR_{\max} = 200$$

$$HR_{\min} = 60$$

$$v_{asc} = .54$$

$$v_{\max} = 3$$

$$VO2_{\max} = 288$$

INITIAL CONDITIONS

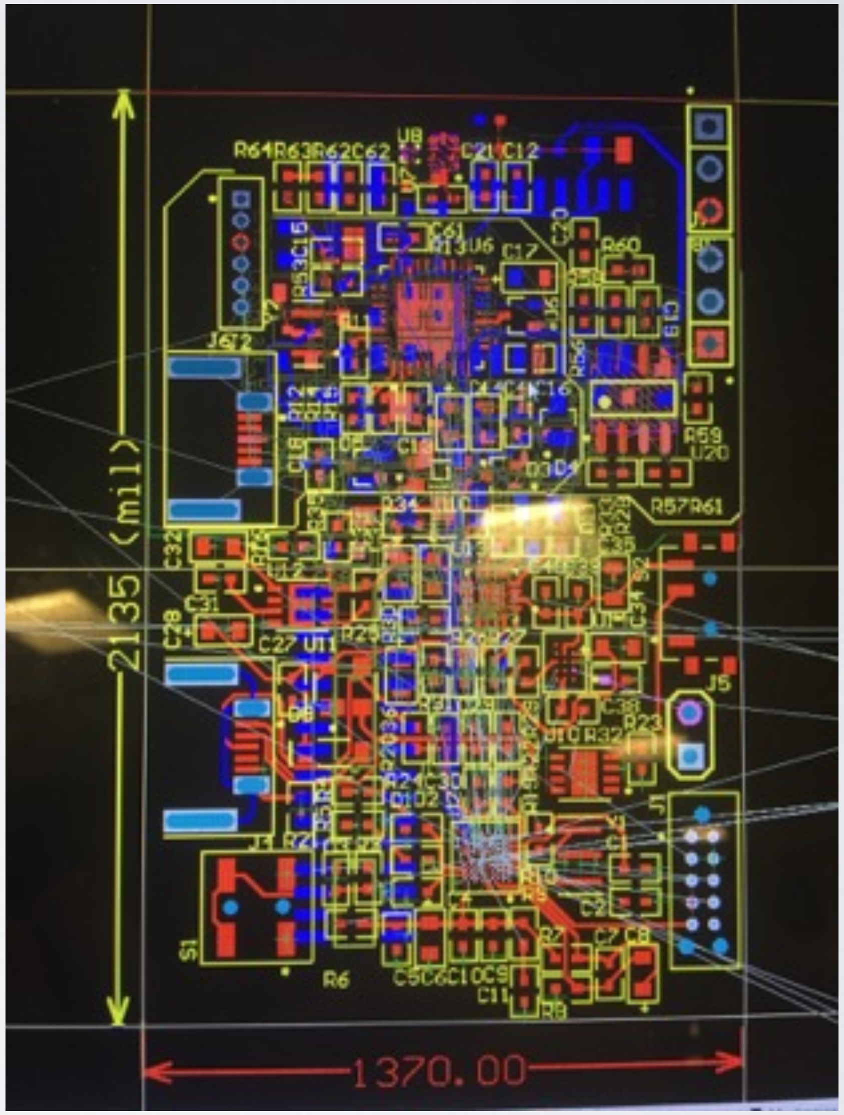
Invariant ... +

$$depth_{\max} = \frac{Tank}{VO2_{\max}} * v_{asc}$$

$$T > 0$$

FUTURE WORK

- Require lots of test data for divers to better fit model parameters
- Prove safety of Maria's (complex) model
- Omar designed a wrist computer for under $300
  - Prototype expected this summer




Thanks!